Section 4.1
Extreme Values of Functions

Absolute Extreme Values

Let f be a function with domain D. Then f(c) is the

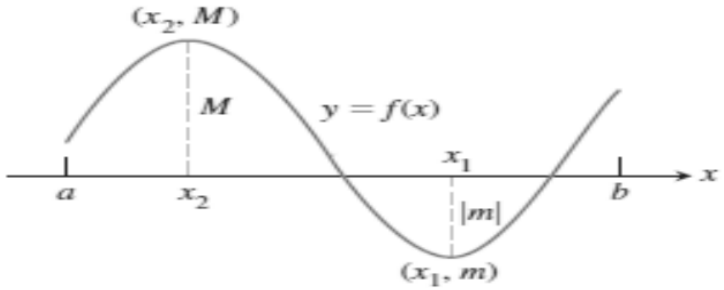
a.) Absolute maximum on D if and only if $f(x) \leq f(c)$ for all x in D

b.) Absolute minimum on D if and only if $f(x) \geq f(c)$ for all x in D

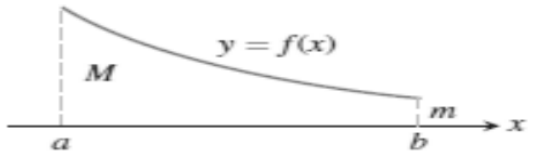
Absolute max and min are called **absolute extrema**

The Extreme Value Theorem

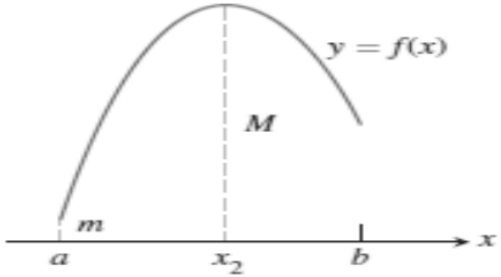
If f is continuous on a closed interval [a,b], then f has both a maximum value and a minimum value on the interval.



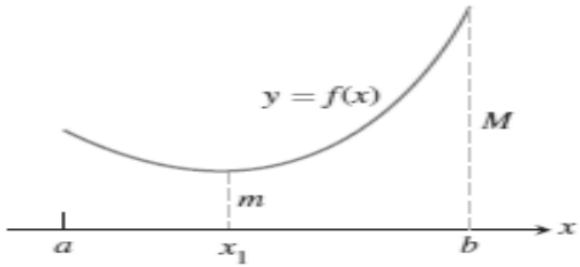
Maximum and minimum at interior points



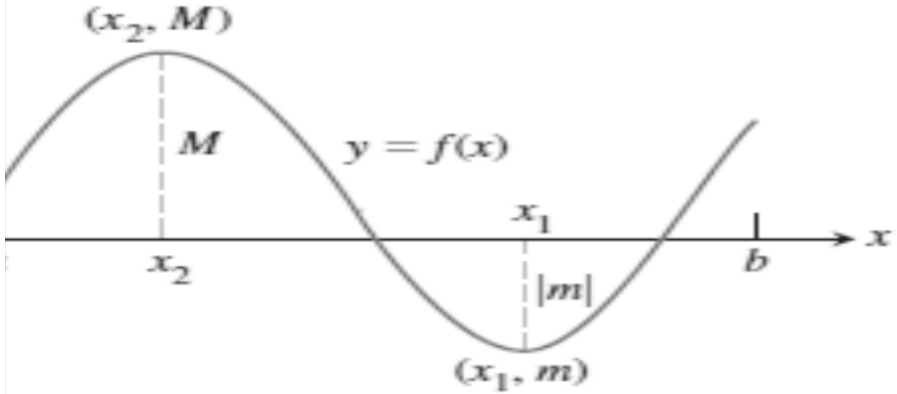
Maximum and minimum at endpoints



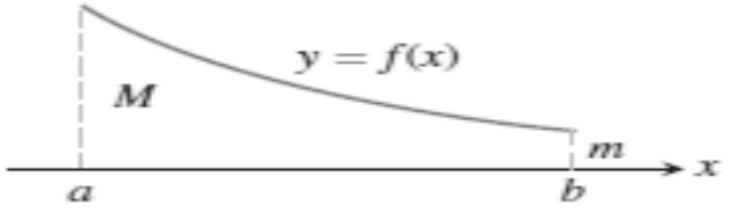
Maximum at interior point, minimum at endpoint



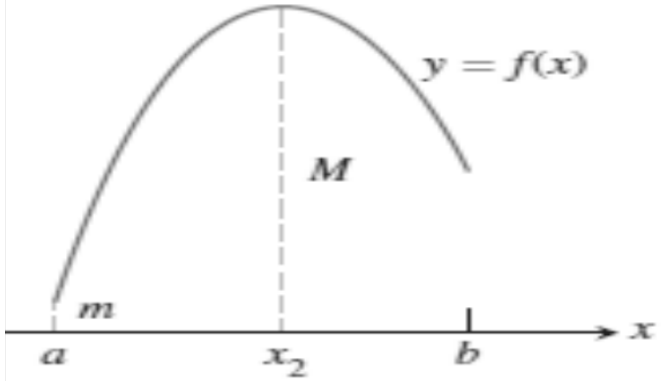
Minimum at interior point, maximum at endpoint



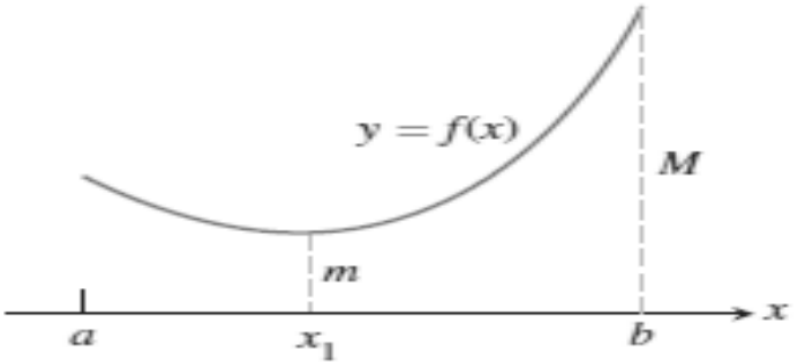
Maximum and minimum at interior points



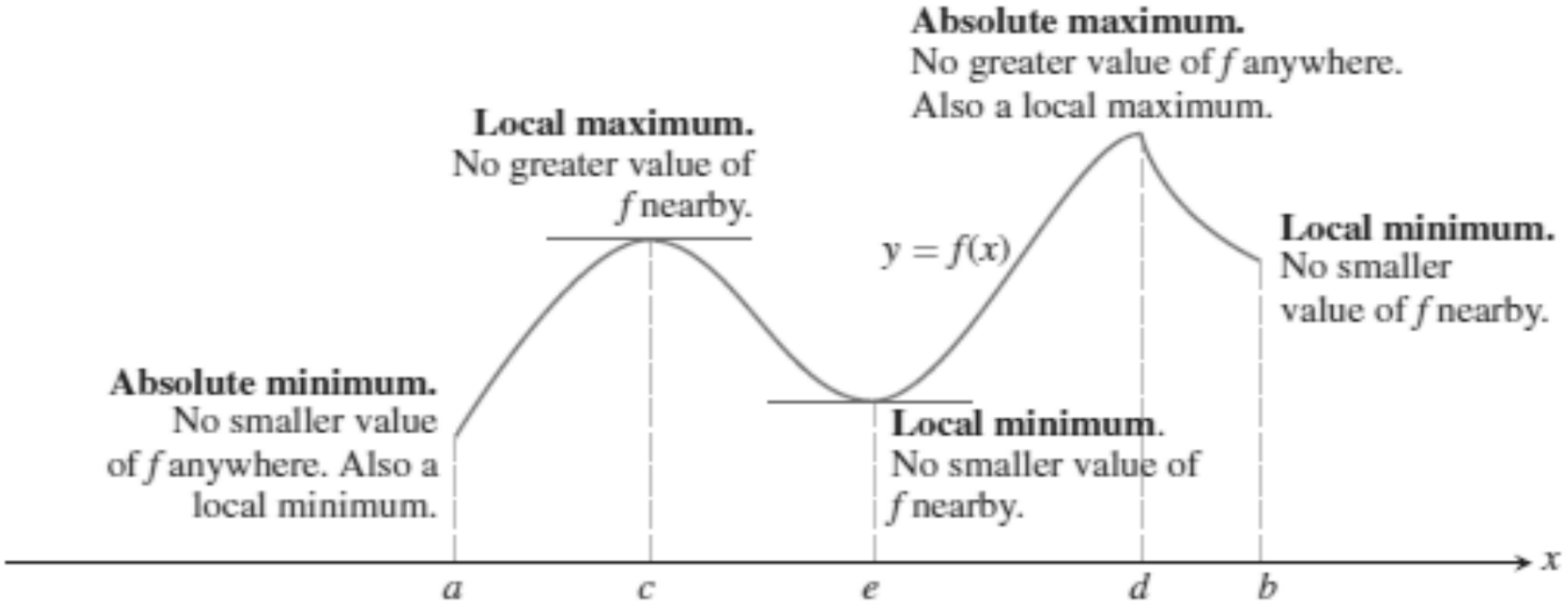
Maximum and minimum at endpoints



Maximum at interior point, minimum at endpoint



Minimum at interior point, maximum at endpoint



When the domain of f(x) is a closed interval, we MUST consider endpoints

Local Extreme Values

Let c be an interior point of the domain of the function f. Then f(c) is a

local maximum value at c if and only if $f(x) \leq f(c)$ for all x in some open interval containing c

local minimum value at c if and only if $f(x) \geq f(c)$ for all x in some open interval containing c

Local Extrema are called relative extrema

Finding Extreme Values

If a function f has a local maximum value or local minimum value at an interior point c of its domain, and if f' exists at c, then

$$f'(c) = 0$$

Critical Point

A point in the interior of the domain of a function f at which f '(x) = 0 or f '(x) does not exist is a critical point of f.

Example

Find the absolute maximum and minimum values of $f(x) = x^{2/3}$ on the interval [-2,3].

$f(-2) = 1.59$

$f(3) = 2.08$ abs. max

$f(0) = 0$ abs. min.

$f'(x) = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x}} = 0$

$2 = 0$

Find the extreme values of $f(x) = \dfrac{1}{\sqrt{4 - x^2}}$

$(-2, 2)$

$$4 - x^2 > 0$$

$$-x^2 > -4$$

$$x^2 < 4$$

$$-2 < x < 2$$

Find the extreme values of

$$f(x) = \begin{cases} 5 - 2x^2, & x \leq 1 \\ x + 2, & x > 1 \end{cases}$$

Left @ $x \leq 1$

$f'(x) = -4x$

$f'(1) = -4$

Right @ $x > 1$

$f'(x) = 1$

$f'(1) = 1$

Find the extreme values of $f(x) = \ln \left| \dfrac{x}{1 + x^2} \right|$